Ocean Waves and wave energy device design

This technical article introduces the description of real seas and indicates how sea characteristics influence the design of wave energy devices.

Description of sea waves
Figure 1a shows a simple sinusoidal transverse wave. This can be imagined to be the cross-section of the wavefront shown in Figure 1b. Waves that look roughly like this can sometimes be seen in real seas, particularly swells, but usually the sea surface is more complex. If we took a cross section through a real sea, it would look more like Figure 2b. The way this diagram was drawn was by adding together several sinusoidal waves that have different **wavelengths** (λ) and different **amplitudes** (a), as illustrated in Figure 2a. We can think of real seas in exactly the same way: the sum of many individual waves that have different wavelengths and different amplitudes. For the purposes of this discussion, let's call the individual waves that make up the sum 'components'.

Figure 1a: Sinusoidal transverse wave

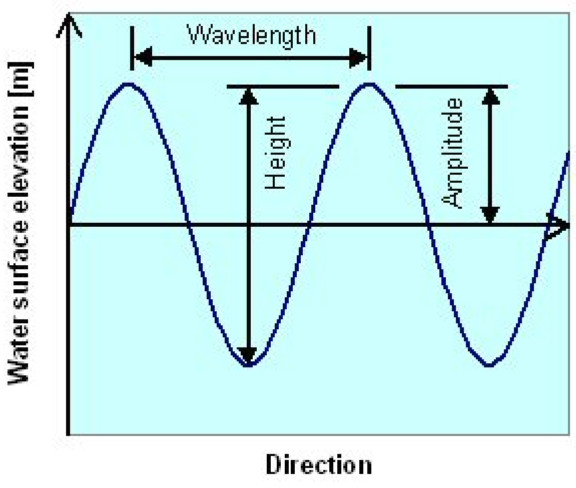


Figure 1b: Sinusoidal transverse wave shown as 3D wave front

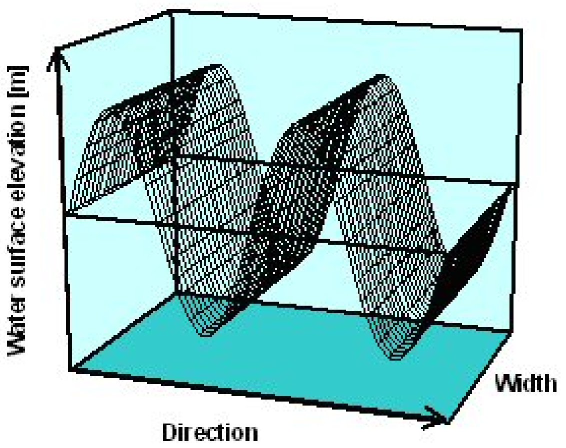


Figure 2a: Several sinusoidal wave components with different amplitudes and wavelengths

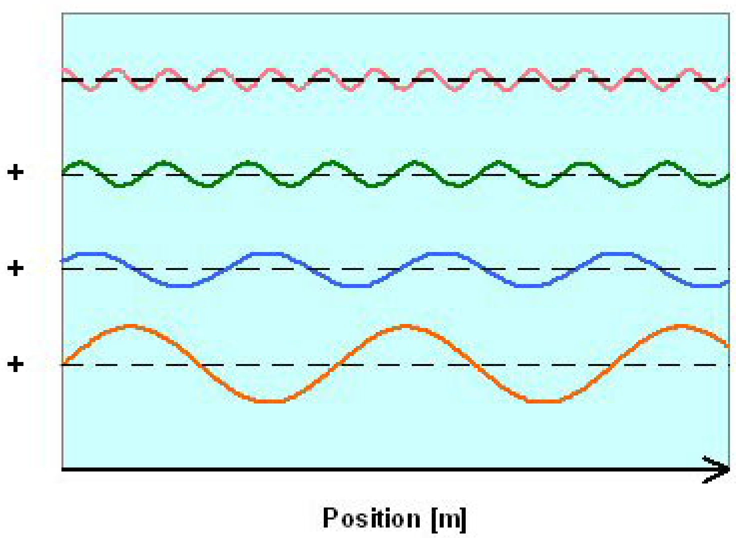
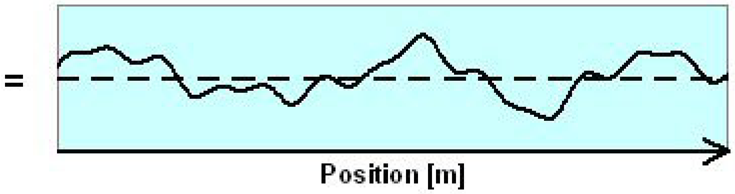


Figure 2b: Result of sum of components shown in Figure 2a



As well as having different wavelengths and different amplitudes, components travel in different directions. When added together, there tends to be a predominant direction corresponding to the direction of greater amplitude and longer period components. On top of this there is some directional 'noise' caused by smaller-amplitude and shorter-period components travelling in other directions. Figures 3a and 3b show the summation of components in three dimensions. Note that these diagrams are similar to Figures 2a and 2b, but the 3D images allow direction to be shown. It can be seen that Figure 3b looks quite like a real sea surface.

Figure 3a: Several sinusoidal wave components with different amplitudes, wavelengths and directions

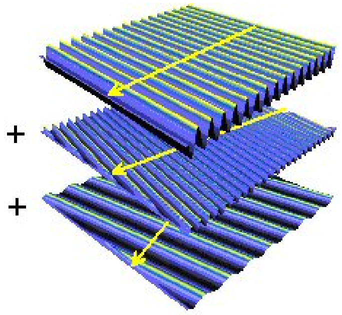


Figure 3b: Result of sum of components shown in Figure 2b

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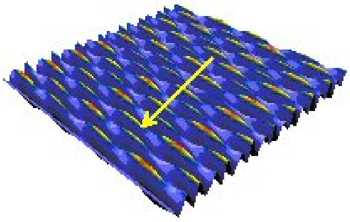
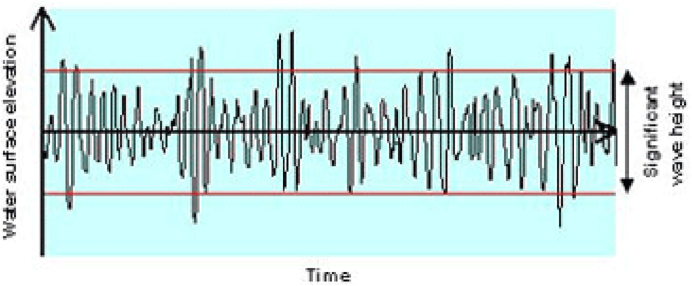


A further parameter we are interested in is the period (T) or frequency (f) of waves. (We can say 'or' because frequency is simply the reciprocal of period: f = 1 / T. If we know the frequency, we immediately know the period.) In deep water, the period of waves is proportional to the square root of the wavelength ($T \propto \sqrt{\lambda}$), and therefore the frequency is proportional to one over the square root of the wavelength ($f \propto 1 / \sqrt{\lambda}$). It is sometimes more convenient to talk about frequency than wavelength because we can readily measure frequency. For example, if we were to place a floating body on the sea surface, we could count how often it heaved up and down during a certain time interval.

Figure 4: Example trace of water surface elevations over time



We also tend to talk about **wave height** (H) in preference to wave amplitude. Figure 1a indicates the meaning of wave height and wave amplitude for a simple sinusoidal wave. But what is the wave height in Figure 4, which shows a trace of the water surface levels over time for a real sea? In practice, we need to adopt a statistical process to describe the wave height of real seas. **Significant wave height** (H_s) was introduced in the technical article as four times the root mean square of water levels relative to the mean water level. In practice, this means:

- Measuring the wave level a certain number of times (n) over a given time interval, with the measurements equally spaced in time;
- Squaring each value of wave level;
- Adding the squared wave levels together; and
- Dividing the total by n, square rooting, and multiplying by four

This gives us a good 'representative' wave height for real seas. Two alternative definitions of wave height are given later in this article.
We might also ask what is the period of waves in Figure 5. A term mentioned in the previous article is **zero up-crossing period** (T_z), which is the average time between successive crossings of the mean water level in an upward direction. This means:

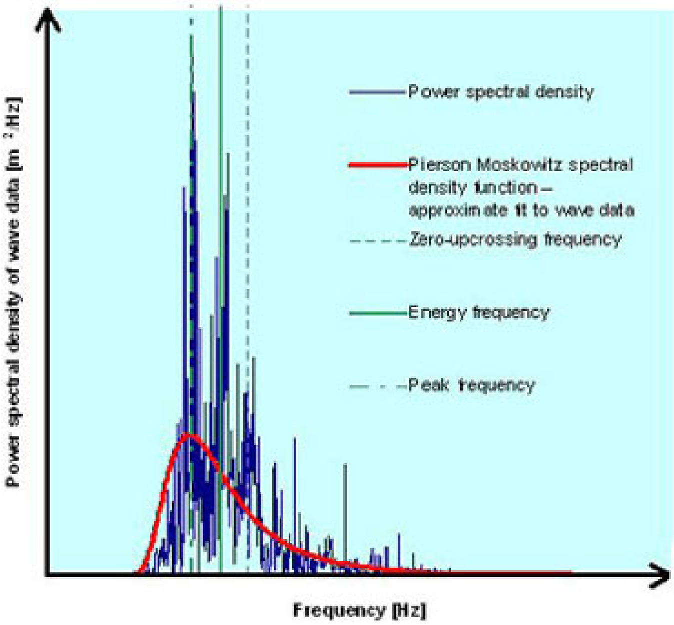
- Counting the number of times the water level crosses the mean water level during the time interval of measurements; and
- Dividing the time interval by this number

Interpretation of wave data
Figure 4 was produced from real sea data that was collected using a **wave rider buoy** . This is a

device that measures wave conditions and its data could be used to determine significant wave height and zero up-crossing period with the statistical processes described. Unfortunately, we cannot 'work backwards' from a trace like Figure 4 to find the components that were added together to form it, i.e. break the wave down into several simple sinusoidal waves. Doing so would tell us more about the energy content.

Instead, we can apply a process called Fourier Transformation. This translates the trace of wave heights over time to a graph of **power spectral density** with respect to frequency, also known as the **wave spectrum** . The graph tells us which frequencies carried most energy in the sea during the time interval of measurements. An example wave spectrum is shown in Figure 5. It can be seen that in this particular sea there is very little energy associated with low-frequency waves and also little energy associated with high-frequency waves. Most of the energy is concentrated in a middle band of frequencies.

Figure 5: Example wave spectrum (power spectral density)

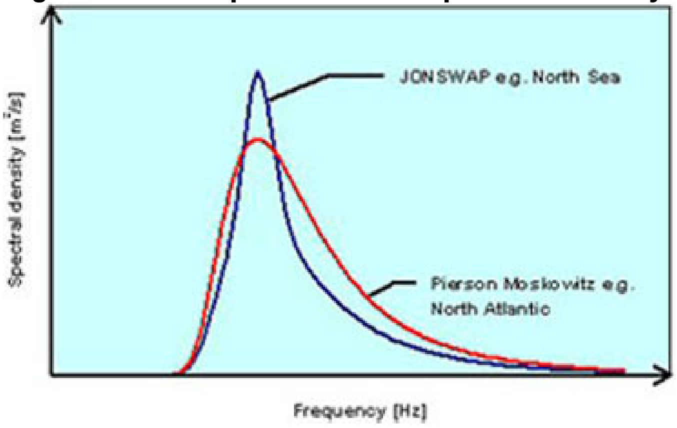


Since we are plotting frequency on the x-axis, we can mark the value of zero up-crossing period (or zero up-crossing frequency) for the wave rider buoy measurements as a vertical line. This falls to the right of the middle band. We can also define two other types of period or frequency:

- **Peak period (T_p) (or peak frequency).** This is the period of the component that has the highest energy content. It coincides with the point of highest spectral density, as illustrated.
- **Energy period (T_e) (or energy frequency).** This is the period of a simple sinusoidal wave that would carry the same energy as the sea. In Figure 5, the energy period lies in the middle of the power spectral density.

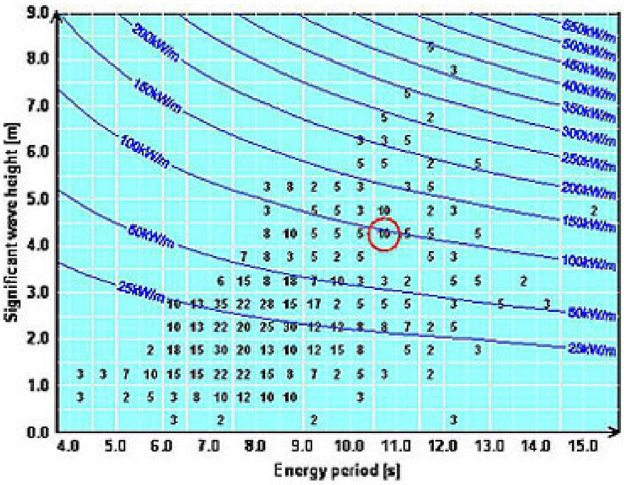
A wave spectrum graph can be produced from data collected by a wave rider buoy at any location, but it turns out that the wave spectra for many locations are similar. We can generalise the shapes of wave spectra by spectral density functions, such as those shown in Figure 6. The Pierson Moskowitz **spectral density function** is typical of locations in the North Atlantic, while the JONSWAP spectral density function is representative of North Sea conditions. (For readers familiar with wind energy, the application of spectral density functions either in the absence of specific site data or to reduce specific site data to a convenient form is analogous to the use of Weibull distributions to describe wind climates.)

Figure 6: Examples of wave spectral density functions



Fourier transformation is often used to summarise long sets of wave data into summary statistics of significant wave height and energy period. We could do this for the whole sea but it is more useful to break the whole sea up into shorter sea states. A sea state is defined by a specific combination of significant wave height and period and in practice specific sea states last for a few hours. All of the examples so far in this article have talked about what happens during a single sea state.
A useful presentation of collected wave data is the scatter diagram. This shows the frequency of occurrence of sea states over a long time interval of measurements. Figure 7 is an example scatter diagram, and includes contours of constant wave power that illustrate the relationships between power (P), wave height and period. In fact, the power in waves is proportional to wave period ($P \propto T$) and wave height squared ($P \propto H^2$). (The scatter diagram is analogous to the wind speed and direction distribution used to describe wind climates.)

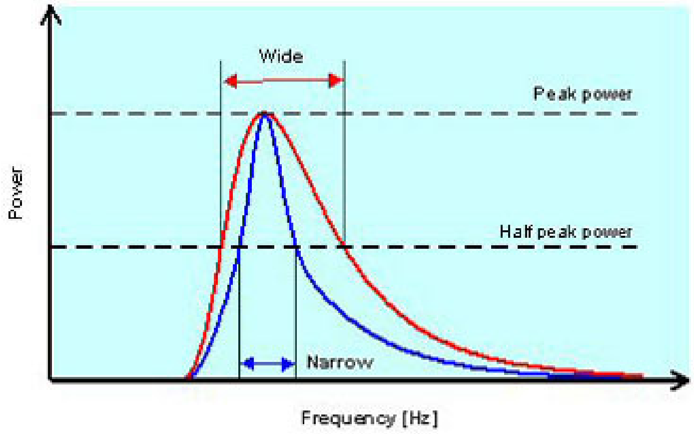
Figure 7: Example scatter diagram showing how often each sea state (combination of significant wave height and zero-up-crossing period) occurs in parts per thousand. The blue lines are contours of constant wave power, and indicate that most sea states carry energy below 50kW/m, but a few rare sea states carry more than 400kW/m. As an example, a single sea state with significant wave height 4.25m and energy period 10.75s is circled in red.



Frequency response of devices and description of power output
In developing wave energy devices we are interested in the criteria that wave conditions set for device design and how descriptions of the sea translate into descriptions of power output. Noting the above distributions of energy with respect to frequency, we might expect that the frequency response of devices is important, and this is true for some types of device. The range of frequencies over which a device captures energy is called its **bandwidth**. Some devices absorb a lot of energy over a narrow band and very little energy outside this band, while other devices absorb energy over

a broader band but less energy at particular frequency within this band. The net energy absorbed may be the same, but the devices may be more or less suited to different sites depending on the sites' wave spectra. Broad-band and narrow-band devices are illustrated in Figure 8.

Figure 8: The power response of two different devices is shown here. The red line indicates a wide bandwidth device and the blue line a narrow bandwidth device. In this case the two devices are compared on their half-peak-power bandwidth. The half-peak-power is the bandwidth between the two frequencies where half of the peak power is produced, shown as dashed lines.



The energy absorption of some devices is greatest when they resonate with the waves. This means that the frequency of the waves is close to the device's natural frequency of oscillation. Since the wave frequency changes over time, the device's natural frequency must also change in order for the device to resonate continuously. Changing a device's natural frequency is known as **tuning**, and may involve adjusting it's size, shape, mass, stiffness or damping, or some combination of these. Tuning can be considered in three contexts, depending on when and how quickly it is done:

- **Fixed tuning** . Properties of the device that it is impossible or at least impractical to change once it is constructed should be set during the design process so that the device's frequency response is a good overall match to the wave spectrum at the intended sea location. These properties are likely to include the device's size, shape and mass;
- **Slow tuning** . Changes in the frequencies of sea waves occur over matters of minutes to hours. In some seas, a particular zero up-crossing period may last for about half an hour. This sets a timeframe for wave devices to change their natural frequency. Tuning a device to match the current wave climate over a period of several minutes to hours is referred to as slow tuning. An example of slow tuning is pumping water in/out of a tank to change the device's buoyancy and therefore its mass and stiffness. Slow tuning allows the device to be better matched to the wave climate over more time than is possible with fixed tuning. It therefore allows more energy to be captured in the long term; and
- **Fast tuning.** While a certain zero up-crossing period may prevail for half an hour, we can tell from Figure 4 that each successive wave has a different height and period. An ideal tuning system would be able to tell the height and period of oncoming waves before they reach the device and adjust the device's properties in advance to extract the maximum possible energy from the waves. This is called fast tuning. In practice, fast tuning is difficult to implement, partly because it is difficult to 'feed forward' wave characteristics to a device and also because it is difficult to change some of the device's properties quickly. But fast tuning may allow more energy to be captured than slow tuning. A device's control system may combine aspects of slow tuning and fast tuning.

Figures 9a and 9b below illustrate the concepts of fixed tuning and slow tuning respectively. In both graphs, the yellow shaded area represents the wave spectrum of the sea. The movements of this area indicate how the wave spectrum changes. The blue curve is the frequency response of the device. In Figure 9a, it can be seen that the device frequency response does not change and only sometimes coincides with the wave spectrum. Figure 9b on the other hand shows the device frequency response following the wave spectrum such that the two always coincide. These illustrations are much simplified and a real wave energy device may adopt some compromise between the two behaviours. For example, it may not be worth forcing the device to respond to high frequency waves if they carry little energy.

Figure 9a: Animation of fixed tuning frequency response

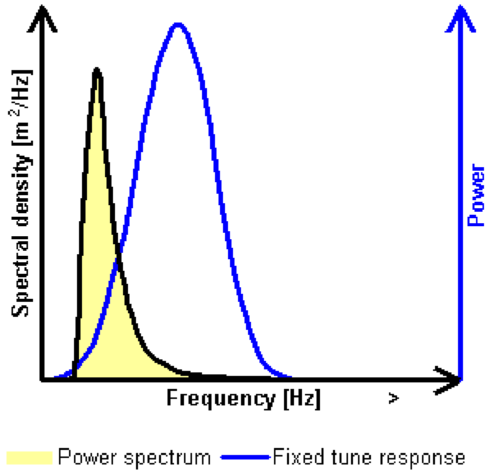
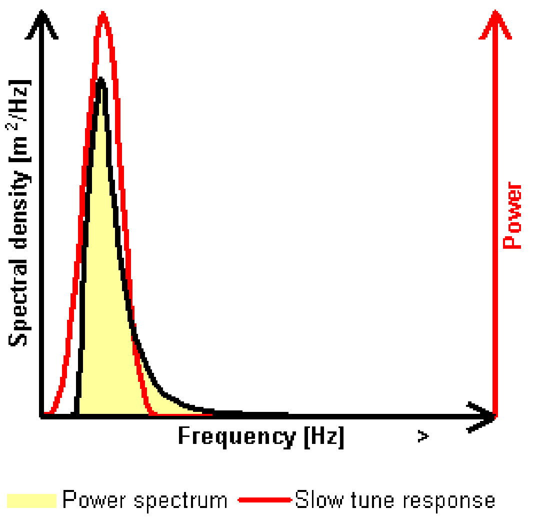


Figure 9b: Animation of slow tuning frequency response



Since the power carried by waves is a function of wave height and period (Figure 7), we might expect the power output of wave energy devices to also be a function of wave height and period. This is true for some devices. The process of calculating power output is complex, and may involve several analytical, numerical and/or empirical models. The end result, however, might look like Figure 10, which is called a **power matrix** or **power surface** . This is similar to the scatter diagram, but instead of indicating power in the sea, the contours show power generated by the device. (The power surface is analogous to the power curve used to describe the generation of wind turbines with respect to wind speed.) If the power surface is combined with the scatter diagram – in particular the occurrences of each sea state – then it is possible to compute the energy output over time. This combination is illustrated in Figure 11.

Figure 10: Example device power surface

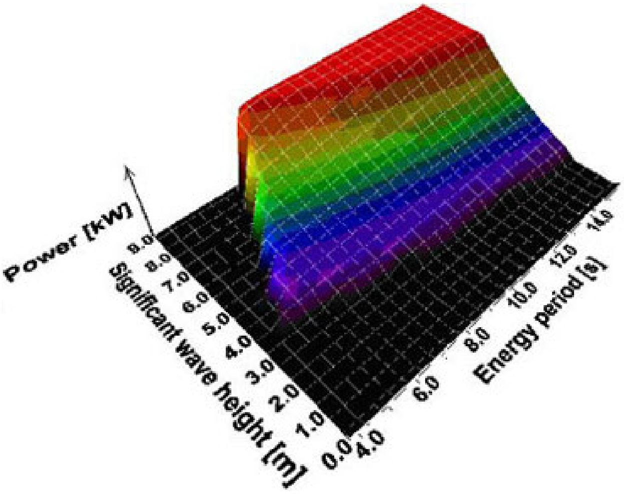


Figure 11: Combination of device power surface and sea scatter diagram